➢ Postulates of special theory of relativity :

I) The fundamental laws of physics have the same form for all inertial system.
II) The velocity of light in vacuum is independent of the relative motion of the source and the observer.

➢ Einstein's mass-energy relation :

From work-energy theorem, the kinetic energy of a moving body is equal to the work done by the external force that impacts the velocity to the body from rest. If 'F' is the force acting on the body, then work done by the force on body in raising its velocity from v = 0 to v = v is given by

$$W = \int_0^v F ds$$

Kinetic energy of the body, $E_k = T = W = \int_0^v F ds$

$$= \int_0^v F \frac{ds}{dt} dt$$

$$= \int_0^v F v dt \qquad \left[v = \frac{ds}{dt} \right]$$

$$= \int_0^v \frac{dp}{dt} v dt \qquad \left[F = \frac{dp}{dt} \right]$$

$$= \int_0^v v \frac{d}{dt}(mv) dt \qquad \left[P = mv \right]$$

$$= \int_0^v v.dmv \qquad \text{---------} \ (1)$$

But from the relation of variation of mass with velocity, the mass of body in motion,

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

Where m_0 is the rest mass of the body.

$$E_k = T = \int_0^v v.d\left(\frac{m_0}{\sqrt{1 - v^2/c^2}} v \right)$$

$$= m_0 \int_0^v v.d\left[v\left(1 - v^2/c^2\right)^{-\frac{1}{2}} \right] \qquad \text{d(uv)=udv+vdu}$$

$$= m_0 \int_0^v v \left[\left(1 - v^2/c^2\right)^{-\frac{1}{2}} dv - v.\frac{1}{2}\left(1 - v^2/c^2\right)^{-\frac{3}{2}} \times \left(-\frac{2v}{c^2} \right) dv \right]$$

$$= m_0 \int_0^v v\left(1 - \frac{v^2}{c^2}\right)^{-\frac{3}{2}} \left[1 - \frac{v^2}{c^2} + \frac{v^2}{c^2}\right] dv$$

$$= m_0 \int_0^v \frac{v\,dv}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}}}$$

$let,$

$$y = 1 - \frac{v^2}{c^2}$$

$$\Rightarrow dy = -\frac{2v\,dv}{c^2}$$

$$\therefore v\,dv = -\frac{c^2}{2} dy$$

v	y
0	1
v	$1 - \frac{v^2}{c^2}$

$$= -m_0 \int_1^{\left(1 - \frac{v^2}{c^2}\right)} \frac{\frac{c^2}{2}}{y^{\frac{3}{2}}} dy$$

$$= \frac{-m_0 c^2}{2} \left[\frac{y^{-\frac{1}{2}}}{-\frac{1}{2}} \right]_1^{\left(1 - \frac{v^2}{c^2}\right)}$$

$$= m_0 c^2 \left[\frac{1}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}} - 1 \right]$$

$$= \left[\frac{m_0}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}} - m_0 \right] c^2$$

$$E = (m - m_0)c^2 \qquad \text{------------------ (2)}$$

This is relativistic expression for kinetic energy. Thus the kinetic energy of a moving body is equal to gain in mass times the square of speed of light.

Therefore m_0c^2 may be regarded as rest energy of a body of rest mass m_0. This rest energy may considered as internal energy of the body.

$\therefore$ Total energy, E = rest energy + relativistic kinetic energy.

$= m_0c^2 + (m - m_0)\, c^2$

$= m_0c^2 + mc^2 - m_0c^2$

$\therefore$ $E = mc^2$

This is Einstein's mass-energy relation.

➢ Problem :

How much electric energy could theoretically be obtained by annihilation of one gram of water?

Solution:

We know,

$$E = mc^2$$
$$= 10^{-3} \text{ x}(3 \times 10^8)^2$$
$$= 9 \times 10^{13} \text{ joule}$$

Here,

$m = 1$ g
$= 10^{-3}$kg
$c = 3 \times 10^8$ ms^{-1}

But, 1ev = 1.602×10^{-19} joule

$\therefore$ Electrical energy obtained = $\dfrac{9\times10^{13}}{1.602\times10^{-19}} ev$

$= 5.618 \times 10^{32}$ev

➢ Photo electric effect :

Ejection of electrons from a metal plate when illumination by light or any other radiation of suitable wavelength or frequency is called photo electric effect.

➢ Einstein photoelectric equation :

Following Planck's idea that light consists of photons Einstein proposed an explanation of photoelectric effect. According to this explanation when a single photon is incident on a metal surface. It is completely absorbed and imparts its energy ***'hf'*** to a single electron.
The photon energy is utilized for two parts-

I) partly for getting the electron free from the atom and away from the metal surface. This energy is known as the photoelectric work function of the metal and is represented by W_0.

II) The balance of the photon energy is used up in giving the electron a kinetic energy of $\frac{1}{2}mv^2$

$\therefore$ hf = $W_0 + \dfrac{1}{2}mv^2$

This is known as Einstein photoelectric equation.

➢ Pauli's Exclusion principle :

It states that in one atom no two electrons can have the same sets of values for the four quantum numbers n, l, m_1 and m_2.

➢ Radioactivity :

The spontaneous emission of powerful radiations exhibited by heavy elements is called radioactivity. Its unit is curie.

There are two types of radioactivity-

I) Natural Radioactivity

II) Artificial Radioactivity

➢ Natural Radioactivity :

Natural radioactivity is that which is exhibited by elements as found in nature. It is always found in heavier elements in the periodic table.

➢ Artificial Radioactivity :

Modern techniques of artificial transmutation of elements have made it possible to produce radioactivity in many other elements much lighter than those that occur in nature. Such type of radioactivity is known as artificial radioactivity.

➢ Radioactivity decay law :

Let us suppose that at the beginning of disintegration i.e. at t=0, the number of radioactive atoms present in a sample is 'N_0'. As time passes, the number of the original atoms decreases due to continuous disintegration. At time't' let the number of atoms of the original kind be 'N'. At this instant, the rate of disintegration (-dN/dt) which is also called activity is proportional to N.

$$-\frac{dN}{dt} \propto N$$

$$-\frac{dN}{dt} = \lambda N \text{ ------------------ (1)}$$

Where λ is the constant of proportionality and is known as the disintegration or decay or radioactive constant.

From (1) $$\frac{\mathrm{dN}}{\mathrm{N}} = -\lambda dt$$

Integrating $$\int_{N_0}^{N} \frac{\mathrm{dN}}{\mathrm{N}} = -\lambda \int_0^t dt$$

$$\left[\log_e N\right]_{N_0}^{N} = -\lambda \left[t\right]_0^t$$

$$\therefore \log_e N - \log_e N_0 = -\lambda t$$
$$\log_e \frac{N}{N_0} = -\lambda t$$
$$\therefore \frac{N}{N_0} = e^{-\lambda t}$$
$$\therefore N = N_0 e^{-\lambda t}$$ Which is radioactive decay law.

➢ Half-life :

Half-life of a radioactive element may be defined as the time during which a given amount of that element is reduced by disintegration to half its initial amount. It is written as $T_{1/2}$.

We know, $N = N_0 e^{-\lambda t}$

For half-life, $N = \frac{N_0}{2}$, $t = T_{1/2}$

So,
$$\frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}}$$
$$\frac{1}{2} = e^{-\lambda T_{1/2}}$$
$$e^{\lambda T_{1/2}} = 2$$
$$\lambda T_{1/2} = \log_e 2$$
$$\lambda T_{1/2} = 0.693$$
$$\therefore T_{1/2} = \frac{0.693}{\lambda}$$

➢ Problem :

A radioactive substance has a half-life of 30 days. Calculate radioactive disintegration constant.

We know, $T_{1/2} = \frac{0.693}{\lambda}$

Here, $T_{1/2}$ = 30 days

$$\therefore \lambda = \frac{0.693}{T_{1/2}}$$
$$= \frac{0.693}{30 days}$$
$$= 0.0231 \text{ Day-1}$$

- **Nuclear Fission :**

The division of a nucleus into two approximately equal parts is called nuclear fission.

Example: ${}_{92}U^{235} + {}_{0}N^{1} \rightarrow {}_{54}Xe^{140} + {}_{38}Sr^{94} + 2{}_{0}n^{1}$ + Energy (200 MeV)

- **Nuclear Fusion :**

It is the process of combining two lighter nuclei into a stable and heavier nuclide.

Example: ${}_{1}^{1}H + {}_{0}n^{1} \rightarrow {}_{1}H^{2}$ + Energy (2.23 MeV)

	Nuclear Fission	Nuclear Fusion
Definition:	Fission is the splitting of a large atom into two or more smaller ones.	Fusion is the fusing of two or more lighter atoms into a larger one.
Natural occurrence of the process:	Fission reaction does not normally occur in nature.	Fusion occurs in stars, such as the sun.
Byproducts of the reaction:	Fission produces many highly radioactive particles.	Few radioactive particles are produced by fusion reaction, but if a fission "trigger" is used, radioactive particles will result from that.
Conditions:	Critical mass of the substance and high-speed neutrons are required.	High density, high temperature environment is required.
Energy Requirement:	Takes little energy to split two atoms in a fission reaction.	Extremely high energy is required to bring two or more protons close enough that nuclear forces overcome their electrostatic repulsion.
Energy Released:	The energy released by fission is a million times greater than that released in chemical reactions, but lower than the energy released by nuclear fusion.	The energy released by fusion is three to four times greater than the energy released by fission.
Nuclear weapon:	One class of nuclear weapon is a fission bomb, also known as an atomic bomb or atom bomb.	One class of nuclear weapon is the hydrogen bomb, which uses a fission reaction to "trigger" a fusion reaction.